CALCULATION OF THE STABILITY OF THE FLOW IN A CIRCULAR PIPE WITH INJECTION

V. M. Eroshenko, L. I. Zaichik,
and V. B. Rabovskii

UDC 532.526

An experimental investigation of the transition of a laminar flow regime into a turbulent one has been carried out in [1] for a flow in a circular pipe which is organized due to injection through the porous lateral surface with a jammed leading end of the pipe. It was established as a result that injection leads to an increase in stability of the laminar flow regime and increases the Reynolds number of the transition to 10,000 instead of the value 2300 which is characteristic of flow in a circular pipe with impenetrable walls. A similar effect was discovered in [2], in which it was also obtained that the Reynolds number of stability loss under the action of injection can take values significantly larger than in pipes with impenetrable walls. The phenomenon of relaminarization of a turbulent flow in the initial section of a circular pipe under the action of injection has been experimentally detected at the entrance for relatively low Reynolds numbers in [3, 4]. Theoretical investigations of stability of flow with injection have been performed only for a plane channel [5, 6]. A calculation is made in this paper of the stability of a hydrodynamically developed flow in a circular pipe with injection through a porous lateral surface.

The stability of a flow against small perturbations is investigated within the framework of linear theory. Just as in [5, 6], the case of two-dimensional perturbations is analyzed on the assumption that stability loss is determined by the properties of the flow near the transverse cross section of the pipe under discussion, i.e., is of a local nature. The linearized equation for the perturbation amplitude in dimensionless form is of the form [7]

$$\left(\frac{d^2}{dr_0^2}-\frac{1}{r_0}\frac{d}{dr_0}-\alpha^2\right)^2\varphi=\frac{\mathrm{Re}}{2}i\alpha\left[(u_{x0}-c)\left(\frac{d^2\varphi}{dr_0^2}-\frac{1}{r_0}\frac{d\varphi}{dr_0}-\alpha^2\varphi\right)-\right.$$
$$\left.-\left(\frac{\partial^2u_{x0}}{\partial r_0^2}-\frac{1}{r_0}\frac{\partial u_{x0}}{\partial r_0}\right)\varphi\right]+u_{r0}\left(\frac{d^3\varphi}{dr_0^3}-\frac{3}{r_0}\frac{d^2\varphi}{dr_0^2}+\frac{4}{r_0^2}\frac{d\varphi}{dr_0}-\alpha^2\frac{d\varphi}{dr_0}+\frac{2\alpha^2}{r_0}\varphi\right)-\left(\frac{\partial^2u_{r0}}{\partial r_0^2}+\frac{1}{r_0}\frac{\partial u_{r0}}{\partial r_0}\right)\frac{d\varphi}{dr_0}. \quad (1)$$

Here the pipe radius r_w is used as the length scale, the average velocity in the cross section under discussion $U = U_0 - 2V_w x/r_w$ is used as the velocity scale, $\mathrm{Re} = 2Ur_w/\nu$ is the Reynolds number of the main flow, V_w is the velocity of the injection ($V_w < 0$), $r_0 = r/r_w$, $u_{x0} = u_x/U$, and $u_{r0} = u_r/U$. The last two terms in Eq. (1) describe the effects of nonparallelness of the flow due to the presence of a radial velocity component.

In the case of injection which is uniform over the pipe length the system of Navier–Stokes equations for the unperturbed motion has the self-similar solution [8]

$$u_x=(U_0-2V_wx/r_w)F'(\eta),\ u_r=V_wF(\eta)/\sqrt{\eta}, \quad (2)$$

where $\eta = r_0^2$. The function $F(\eta)$ is found from the solution of the ordinary differential equation

$$(\eta F'')''+(R/2)(F'F''-FF''')=0,\ R=V_wr_w/\nu \quad (3)$$

with the boundary conditions

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 3, pp. 82-86, May-June, 1984. Original article submitted April 11, 1983.

 0021-8944/84/2503-0408$08.50

$$F(1)=1,\quad F'(1)=0,\quad \lim_{\eta\to 0} F/\sqrt{\eta}=0,\quad \lim_{\eta\to 0}\sqrt{\eta}F''=0. \tag{4}$$

Switching in (1) to the variable η, we obtain with the expressions (2) taken into account the following equation for the perturbation amplitude:

$$(\eta\varphi'')''-2\alpha_1^2\varphi''+\alpha_1^4\varphi/\eta=i\alpha_1\,\mathrm{Re}_1\left[(F'-c)\left(\varphi''-\alpha_1^2\frac{\varphi}{\eta}\right)-F'''\varphi\right]+ \\ +\mathrm{R}_1\left\{F\left[\varphi'''-\alpha_1^2\left(\frac{\varphi}{\eta}\right)'\right]-F''\varphi'\right\}, \tag{5}$$

where $\mathrm{Re}_1 = \mathrm{Re}/4$, $\mathrm{R}_1 = \mathrm{R}/2$, and $\alpha_1 = \alpha/2$. The boundary conditions for Eq. (5) are specified in the form

$$\lim_{\eta\to 0}\varphi/\sqrt{\eta}=0,\quad \varphi'(0)=\mathrm{const},\quad \varphi(1)=\varphi'(1)=0. \tag{6}$$

Equation (5) was replaced by the system of equations

$$\eta\varphi''=\eta\theta+\alpha_1^2\varphi, \\ (\eta\theta)''=\left[\alpha_1^2+i\alpha_1\,\mathrm{Re}_1(F'-c)\right]\theta-i\alpha_1\,\mathrm{Re}_1F'''\varphi+\mathrm{R}_1F\theta'-\mathrm{R}_1F''\varphi'. \tag{7}$$

for the numerical integration. The boundary conditions for Eqs. (7) with (4) and (6) taken into account were specified in the form

$$\varphi(0)=0,\quad \theta'(0)=\frac{\alpha_1^2+i\alpha_1\,\mathrm{Re}_1[F'(0)-c]}{2}\theta(0)-\frac{\mathrm{R}_1F''(0)}{2}\varphi'(0), \\ \varphi(1)=\varphi'(1)=0. \tag{8}$$

The eigenvalue problem for (7) and (8) was solved by the differential sweep method described in [9]. The neutral curves α(Re) and c(Re) for a specified R (Fig. 1) are found from its solution, and were obtained in the range of variation of the injection parameter $-\infty < \mathrm{R} < 52.6$. As $\mathrm{R}\to -52.6$ the critical Reynolds number Re_* increases without limit, and α_* tends to zero. Thus in the case of inflow with $-52.6 < \mathrm{R} < 0$ a developed laminar flow in a circular pipe turns out, just as does a Poiseuille flow, to be stable against small perturbations and can theoretically exist for as large Reynolds numbers as desired. As $|\mathrm{R}|$ increases the value of Re_* declines, reaches a minimum at $|\mathrm{R}| > 120$, and then starts to increase; for $|\mathrm{R}| > 200$ it does so according to a law which is close to linear. For large injections the region of increasing perturbations, which is contained within the $\alpha(\mathrm{Re}_1)$ neutral curve, becomes open (the upper branch of the neutral curve increases without limit as Re_1 increases). One should note that the lower branch of the $c(\mathrm{Re}_1)$ curve corresponds to the upper branch of the $\alpha_1(\mathrm{Re}_1)$ neutral curve. Consequently, in this case short-wavelength perturbations propagate with a lower velocity than do the long-wavelength ones, in contrast to the situation for a Poiseuille flow in a plane channel and a Blasius flow in a boundary layer. The critical value of the Strouhal number $\mathrm{Sh}_* = \alpha_*c_*U/(2\pi|V_w|)$ for $|\mathrm{R}| > 200$ takes the practically constant value $\mathrm{Sh}_* \approx 7$. The value obtained for the Strouhal number is in good agreement with the experimental value [2] $\mathrm{Sh}_m = \alpha c U_m/(2\pi|V_w|) \sim 11$ (U_m is the maximum velocity in a specified cross section), since $\mathrm{Sh}_m/\mathrm{Sh} = U_m/U \approx \pi/2$.

In the limiting case of strong injection as $\mathrm{R}\to -\infty$ the stability parameters of the flow under discussion can be calculated, just as in the case of a plane flow near the front point of a cylindrical body [10], from the solution of the problem without account taken of the viscous terms in Eq. (5), i.e., from the solution of the equation

$$i\alpha_1\Omega\left[(F'-c)\left(\varphi''-\alpha_1^2\varphi/\eta\right)-F'''\varphi\right]-F\left[\varphi'''-\alpha_1^2(\varphi/\eta)'\right]+F''\varphi'=0, \tag{9}$$

where $\Omega = -\mathrm{Re}/2\mathrm{R} = -U/V_w$.

The velocity function F(η) in Eq. (9) is specified from an implicit limiting solution of Eq. (3) and is of the form

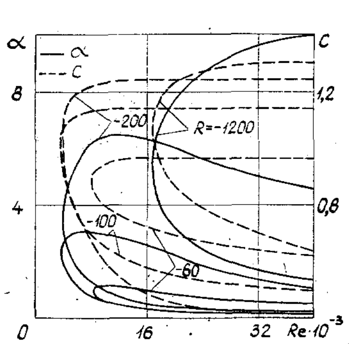

Fig. 1

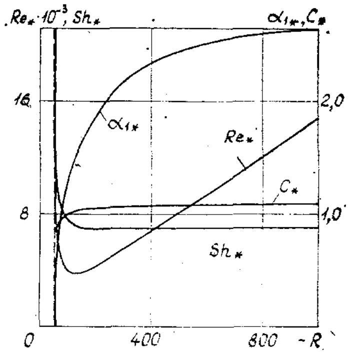

Fig. 2

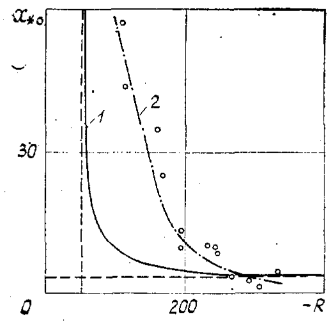

Fig. 3

$$F = \sin(\pi\eta/2). \tag{10}$$

It is interesting to note that the solution of the third-order differential equation (9) satisfies all four boundary conditions (6), just as the solution for the main flow (10) satisfies all the boundary conditions (4). Due to this fact, the solution of Eq. (9) is uniform over the cross section of the pipe as a limit to the solution of the complete equation (5) as $R \to -\infty$. The values of the critical parameters determined from Eq. (9) are equal to: $\Omega_* = 6.84$, $\alpha_{1*} = 2.94$, and $c_* = 1.08$. Thus the mechanism of stability loss in a circular pipe with strong injection is of a nonviscous nature.

A comparison of the results of a calculation of the cross section in which the loss of stability of the flow against small perturbations occurs (curve 1) with the experimental data at the onset of the transition [2] (open circles and curve 2) is shown in Fig. 3. The point of stability loss was determined on the basis of the dependence of Re_* on R from the relationship

$$x_{*0} = x_*/r_w = \mathrm{Re}_*/4\,|\,\mathrm{R}\,|.$$

The satisfactory agreement between the experimental and computational data for strong injection indicates a closeness of the coordinates of the points of stability loss and the onset of the transition of laminar flow to turbulent flow. As the strength of the injection increases, a monotonic decrease of the value of x_{*0} occurs, as is evident from Fig. 3, which tends to $\Omega_*/2$. Consequently, notwithstanding the fact that a strong injection leads to an increase in the stability of the laminar flow mode in the sense of an increase of Re_*, the length of the region of laminar flow organized by an injection of liquid through a penetrable lateral surface decreases as the Reynolds number of the injection increases, as in a plane channel [5]. However, on the section $x_0 < \Omega/2$ a flow can exist which is stable against small perturbations for as large values of $|R|$ as desired.

In conclusion we shall estimate the validity of the locally uniform method of solution of the equations used for small perturbations. The approximation under discussion is competent in the case in which the wavelength of the perturbations is less than the nonuniformity scale of the flow, i.e., when $|dU/dx|\ r_w/(\alpha U) = 2/(\Omega\alpha) \ll 1$. It follows from the computational results that the quantity $2/(\Omega\alpha)$ for the critical point reaches its maximum value

$((2/(\Omega\alpha))_{max} \approx 0.1)$ with a strong injection ($R \to -\infty$). Consequently, the locally uniform approximation is well satisfied for all injection strengths.

LITERATURE CITED

1. K. Huesmann and E. R. G. Eckert, "Investigations of a laminar flow and the transition to turbulence in porous pipes with symmetric injection through the pipe wall," Wörme und Stoffubertragung, 1, No. 1 (1968).
2. V. I. Yagodkin, "The use of channels with porous walls for the investigation of intra-channel combustion of solid rocket fuels," in: Transactions of the 18th International Astronautical Congress [in Russian], Belgrade (1967).
3. E. R. G. Eckert and W. Rodi, "Reverse transition for turbulent-laminar flow through a tube with fluid injection," J. App. Mech., 35, No. 4 (1968).
4. W. T. Pennel, E. R. G. Eckert, and E. M. Sparrow, "Laminarization of turbulent pipe flow by fluid injection," J. Fluid Mech., 52, No. 3 (1972).
5. V. N. Varapaev and V. I. Yagodkin, "The stability of the flow in a channel with penetrable walls," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 5 (1969).
6. V. N. Varapaev, N. A. Kuril'skaya, et al., "The stability of non-self-similar flows in channels with penetrable walls," Tr. MISI, No. 102 (1973).
7. F. C. T. Shen, T. S. Chen, and L. M. Huang, "The effects of mainflow radial velocity on the stability of developing laminar pipe flow," J. Appl. Mech., 43, No. 2 (1976).
8. R. M. Terrill and P. W. Thomas, "On laminar flow through a uniformly porous pipe," Appl. Scient. Res., 21, No. 1 (1969).
9. M. A. Gol'dshtik and V. N. Shtern, Hydrodynamic Stability and Turbulence [in Russian], Nauka, Novosibirsk (1977).
10. V. M. Eroshenko, L. I. Zaichik, and V. B. Rabovskii, "The stability of a plane flow near a leading critical point with strong injection," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 2 (1982).

NONLINEAR AZIMUTHAL WAVES IN A CENTRIFUGE

A. A. Abrashkin

UDC 532.59

Azimuthal wave motions in a liquid which partially fills a cylinder (centrifuge) rapidly rotating about a horizontal axis are discussed in this paper. Under the action of centrifugal force the liquid is pressed to the wall of the cylinder and moves together with it about the central air core. The vibrations of the free surface which arise are called centrifugal waves [1]. The difficulties of their theoretical investigation are related to the nonlinearity both of the basic equations and also of the boundary condition for the pressure on the free surface; therefore they have previously been studied only by linear methods [1, 2]. Nonlinear azimuthal waves in a centrifuge with an infinite radius of the rotating cylinder are analytically described below. The waves found are an analog of Gerstner trochoidal waves on a cylindrical surface. An approximate solution for a centrifuge with a finite outer radius is constructed by matching the waves obtained to the known linear ones.

1. We shall consider azimuthal waves in a centrifuge rotating at a constant angular velocity Ω. They have been investigated in the linear approximation in [2]. In the polar coordinate system R, θ rotating with velocity Ω the radial u and azimuthal v velocities are equal, respectively, to

Gor'kii. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 3, pp. 86-89, May-June, 1984. Original article submitted February 24, 1983.

0021-8944/84/2503-0411$08.50